CHAPTER 10

On the Interplay between Measurable and Topological Dynamics

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Introduction

Recurrent–wandering, conservative–dissipative, contracting–expanding, deterministic–chaotic, isometric–mixing, periodic–turbulent, distal–proximal, the list can go on and on. These (pairs of) words—all of which can be found in the dictionary—convey dynamical images and were therefore adopted by mathematicians to denote one or another mathematical aspect of a dynamical system.

The two sister branches of the theory of dynamical systems called *ergodic theory* (or *measurable dynamics*) and *topological dynamics* use these words to describe different but parallel notions in their respective theories and the surprising fact is that many of the corresponding results are rather similar. In the following chapter we have tried to demonstrate both the parallelism and the discord between ergodic theory and topological dynamics. We hope that the subjects we chose to deal with will successfully demonstrate this duality.

The table of contents gives a detailed listing of the topics covered. In the first part we have detailed the strong analogies between ergodic theory and topological dynamics as shown in the treatment of recurrence phenomena, equicontinuity and weak mixing, distality and entropy. In the case of distality the topological version came first and the theory of measurable distality was strongly influenced by the topological results. For entropy theory the influence clearly was in the opposite direction. The prototypical result of the second part is the statement that any abstract measure probability preserving system can be represented as a continuous transformation of a compact space, and thus in some sense ergodic theory embeds into topological dynamics.

We have not attempted in any way to be either systematic or comprehensive. Rather our choice of subjects was motivated by taste, interest and knowledge and to great extent is random. We did try to make the survey accessible to non-specialists, and for this reason we deal throughout with the simplest case of actions of $\mathbb{Z}$. Most of the discussion carries over to non-invertible mappings and to $\mathbb{R}$ actions. Indeed much of what we describe can be carried over to general amenable groups. Similarly, we have for the most part given rather complete definitions. Nonetheless, we did take advantage of the fact that this chapter is part of a handbook and for some of the definitions, basic notions and well known results we refer the reader to volume I of this handbook, mainly to Chapters 1, by B. Hasselblatt and A. Katok, and 2, by J.-P. Thouvenot. Finally, we should acknowledge the fact that we made use of parts of our previous expositions [87] and [36].

We made the writing of this survey more pleasurable for us by the introduction of a few original results. In particular the following results are entirely or partially new. Theorem 1.2 (the equivalence of the existence of a Borel cross-section with the coincidence of recurrence and periodicity), most of the material in Section 4 (on topological mild-mixing), all of Subsection 7.4 (the converse side of the local variational principle) and Subsection 7.6 (on topological determinism).