

MATH 312H:
FUNDAMENTAL STRUCTURES OF CONTINUOUS MATHEMATICS

SPRING 2004

A.Katok

MIDTERM EXAMINATION

February 27, 6:15 pm.

Write a detailed solution for each of the problems below. After you've read and understood the problems you have three hours to do that in one stretch at any time between now and Monday morning. Please bring written solutions to the class on Monday March 1.

You may use results discussed in class in your solutions. For the external material you may use your own course notes only. You may not consult with anyone.

If you have any questions please e-mail me immediately *before* you start the clock.

Good luck!

M1. Prove that the set of all polynomials with rational coefficients is countable.

M2. Consider the set S of all infinite sequences $x_1, x_2, \dots, x_n \dots$ where the n th term x_n can take n values, say $\{1, \dots, n\}$. Prove that the set S has the power of continuum.

M3. Let $a_n, n = 1, 2, \dots$ be a Cauchy sequence of rational numbers. Prove that $\frac{3a_n^3}{1+a_n^2}$ is also a Cauchy sequence.

M4. Let $a_n, n = 1, 2, \dots$ and $b_n, n = 1, 2, \dots$ be two Cauchy sequences of rational numbers. Denote $c_n = \max\{a_n, b_n\}$. Prove that $c_n, n = 1, 2, \dots$ is also a Cauchy sequence.