MATH 312H:
FUNDAMENTAL STRUCTURES OF CONTINUOUS MATHEMATICS

SPRING 2004

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PROBLEM LIST #2 :

Problems on this list are designed for various purposes: Those marked with *) are homework problems; written solutions are due on the date indicated. Unmarked problems usually will be discussed in class; you should give those problems some thought beforehand. Some of those later may be designated as homework. Problems marked **) are more advanced and optional; both solutions and questions in class or by email about those problems are welcome.

8*).Write down a careful proof of the following statement: If A is any uncountable set and $C \subset A$ a finite or countable subset, then A and $A \setminus C$ have the same power.

Due on Monday January 26.

9*).Prove that the set of all *finite* subsets of a countable set is countable.

Due on Monday February 2.

10.Prove that the set of all finite subsets of a continuum has the power of continuum. Give a geometric interpretation of this fact.

11.Prove that the set of all *countable* subsets of a continuum has the power of continuum.

Hint: Think of the method used to prove countability of rational numbers (The *first* Cantor diagonal process).

12.Prove that the set of all subsets of a continuum has power greater than continuum.

Hint: Think of the method used to prove uncountability of continuum (The *second* Cantor diagonal process).

13.Prove that the set of all real–valued functions on the real line has the same power as the set of all subsets of real line.

Hint: It is enough to prove that each of the two sets has the same power as a subset of the other.

14**).Prove that the set of all *continuous* functions on the real line has the power of continuum.