

**MATH 312H:**  
**FUNDAMENTAL STRUCTURES OF CONTINUOUS MATHEMATICS**

SPRING 2004

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PROBLEM LIST # 3:

Problems on this list are designed for various purposes: Those marked with \*) are homework problems; written solutions are due on the date indicated. Unmarked problems usually will be discussed in class; you should give those problems some thought beforehand. Some of those later may be designated as homework. Problems marked \*\*) are more advanced and optional; both solutions and questions in class or by email about those problems are welcome .

15. *Greek method for finding square roots.* Let  $a > 1$  be a rational number. Define two sequences  $a_n$  and  $b_n$  recursively:

$$a_1 = a, \quad b_1 = 1, \quad a_{n+1} = \frac{a_n + b_n}{2}, \quad b_{n+1} = \frac{2a_n b_n}{a_n + b_n}.$$

Prove that both sequences are Cauchy sequences, that they are equivalent and that sequences  $a_n^2$  and  $b_n^2$  converge to  $a$ .

16. \*) Justify the standard rules for taking limits of the sum, product and ratio for sequences of real numbers using the definition of real numbers as equivalence classes of Cauchy sequences of rationals.

*Due on Monday March 15.*

17. \*) Prove that monotone increasing sequence bounded from above is Cauchy.

*Due on Monday March 15.*