MATH 312H:
FUNDAMENTAL STRUCTURES OF CONTINUOUS MATHEMATICS

SPRING 2004

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PROBLEM LIST # 4:

Written solution to the problems on this list are due in the dates indicated.

18.*) Consider Euclidean metric on the plane $\mathbb{R}^2$: For $p_1 = (x_1, y_1)$ and $p_2 = (x_2, y_2)$

$$d(p_1, p_2) = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}.$$

Prove that equality $d(p_1, p_2) = d(p_1, P - 3) + d(p_2, p_3)$ takes place if and only if $p_3 = tp_1 + (1 - t)p_2$ for some t, $0 \le t \le 1$.

Due on Wednesday, March 31.

19.*) Prove that $\mathbb{R}^2$ is a complete metric space with respect to the Euclidean metric

Due on Wednesday, March 31.

20.*) Prove that every isometry of the real line $\mathbb{R}$ with the standard absolute value metric $d(x, y) = |x - y|$ is either a translation $x \to x + t$ or a reflection $x \to -x + t$ for some $t \in \mathbb{R}$.

Due on Friday, April 2.