

Differentiable rigidity of group actions

Summer School, July 7 – July 19, Bedlewo, 2008

Lecture 1

- Anosov maps and flows. Structural stability. Foliations.
- Group actions, \mathbb{Z}^k and \mathbb{R}^k actions, Anosov actions of higher rank Abelian groups. Differentiable rigidity.
- Hirsh-Pugh-Shub theorem about structural stability of central foliations.

Lecture 2

- Resonances. Normal forms. Poincare-Dulac theorem. Sternberg-Chen theorem.
- Sub-resonances. Normal forms of extensions of continuous maps.
- Coarse Lyapunov decomposition for algebraic actions.

Lecture 3

- The proof of differential rigidity of Anosov actions of higher rank Abelian groups:
 1. Hölder conjugacy.
 2. Invariance of weak stable foliations.
 3. Construction of a perturbed action and natural extension.
 4. Smoothness along coarse Lyapunov directions.
 5. Global smoothness.

Lecture 4

- Rotations on the circle, cocycles over rotations, small divisors.
- Circle diffeomorphisms close to rotations, Diophantine rotation number.
- Commuting circle diffeomorphisms close to rotations, simultaneously Diophantine condition.

Lecture 5

- Actions by commuting ergodic toral automorphisms: existence.
- Cocycles over such actions.
- Small perturbations and analysis of the conjugacy problem.

Lecture 6

- Almost cocycles are approximable by cocycles.
- Proof of the local rigidity for actions by commuting ergodic toral automorphisms.
- General outline of applying the KAM iteration to local rigidity of algebraic group actions.