

## Harmonic Analysis, Number theory and Dynamics

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### Lecture 1: Introduction to Szemerédi's theorem

- Background and history
- The idea: randomness vs. structure
- Discrete Fourier analysis
- Proof of Roth's theorem

### Lecture 2: Szemerédi's theorem for $k > 3$

- Higher order uniformity, Gowers norms
- Nonuniformity and polynomial correlations
- Inverse theorems: weak and strong
- Quadratic Fourier analysis

### Lecture 3: The Green-Tao theorem

- Background and history
- The transference principle for the primes
- The main decomposition: random and almost periodic components
- The Furstenberg tower (energy increment argument)

### Lecture 4: Further developments

- Hypergraphs and the multidimensional Szemerédi theorem
- The Bergelson-Leibman and Tao-Ziegler theorems
- The Green-Tao work on the Dickson conjecture
- Quadratic Fourier analysis, apres Gowers
- more, if time permits

