RAPID EVOLUTION OF COMPLEX LIMIT CYCLES

NIKOLAY DIMITROV

Limit cycles of planar polynomial vector fields have long been a focus of extensive research. Analogous to the real case, similar problems have been studied in the complex plane where a polynomial differential one-form gives rise to a foliation by Riemann surfaces. In this setting, a marked cycles is defined as a nontrivial element of the fundamental group of a leaf from the foliation. Whenever the polynomial foliation comes from a perturbation of an exact one-form, one can introduce the notion of a marked multi-fold cycle. This is a marked cycle with at least one representative that determines a free homotopy class of loops in an open fibered subdomain of the complex plane. The topology of this subdomain is closely related to the exact one-form, mentioned earlier. This talk will be an introduction to the notion of marked multi-fold cycles of a close-to-integrable polynomial foliation. We will explore the way they correspond to periodic orbits of the Pioncaré map of the foliation. We will also discuss the "rapid evolution of limit cycles" - the tendency of a continuous family of marked multi-fold limit cycles to escape from certain large open domains in the complex plane as the foliation converges to its integrable part.