

MEASURE RIGIDITY.

Lectures 1 and 2. Introduction and \mathbb{Z}^2 actions by automorphisms of \mathbb{T}^3 .

- Furstenberg problem, algebraic toral examples.
- Suspension construction.
- Lyapunov exponents, Weyl chambers, Lyapunov distributions and foliations.
- Cartan actions and TNS actions.
- Pesin sets and invariant manifolds.
- Conditional measures and entropy (Ledrappier-Young results).
- Measure rigidity for the algebraic examples on \mathbb{T}^3 .

Lectures 3 and 4. Non-uniformly hyperbolic actions of \mathbb{Z}^k on \mathbb{T}^{k+1} , $k \geq 2$.

- Existence of the semiconjugacy.
- Preservation of Weyl chambers under the semiconjugacy.
- Invariant affine structures on the leaves of Lyapunov foliations.
- Uniform growth estimates along the walls of Weyl chambers.
- Ergodicity along the walls of Weyl chambers (π -partition trick).
- Invariance properties of conditional measures and absolute continuity.

Lectures 5 and 6. Non-uniformly hyperbolic actions on arbitrary manifolds.

- Lyapunov metric and special time change (synchronization).
- Properties of the time change and invariant “manifolds”.
- π -partition trick.
- Absolute continuity of conditional measures and conclusion of the proof.
- Further remarks and open questions.

If time permits we shall also discuss uniqueness of the large measure in the non-uniform toral case and other related topics.

References:

- (1) B. Hasselblatt and A. Katok ”Introduction to the Modern Theory of Dynamical Systems”, Encyclopedia of Mathematics and Its Applications 54 Cambridge University Press, 1995. ISBN 0-521-34187-6
- (2) A. Katok and R. Spatzier, ”Invariant measures for higher rank hyperbolic Abelian actions”, *Erg. Theory and Dynam. Systems*, 16 (1996), 751-778,
- (3) A. Katok and B. Kalinin, ”Invariant measures for actions of higher rank abelian groups”, in *Smooth Ergodic Theory and its applications*, Proc. Symp. Pure Math., 69 (2001), 593–637
- (4) A. Katok and B. Kalinin, ”Measure rigidity beyond uniform hyperbolicity: Invariant Measures for Cartan actions on Tori”, *Journal of Modern Dynamics*, 1, N1, (2007), 123–146
- (5) A. Katok and F. Rodriguez Hertz, ”Uniqueness of large invariant measures for \mathbb{Z}^k actions with Cartan homotopy data”, *Journal of Modern Dynamics*, 1, N2, (2007), 287–300
- (6) A. Katok, B. Kalinin, and F. Rodriguez Hertz ”Nonuniform measure rigidity”. (Preprint)