Lectures 1 and 2 (180 min)

1. Introduction
   - Cohomological equations, origins, toral example. Gottschalk-Hedlund's theorem
   - Stability, rigidity, the Greenfield-Wallach-Katok conjecture.
   - Invariant distributions as obstructions. Interpretation of the diophantine condition in the toral example as existence of a bounded inverse $X^(- 1)$for Sobolev functions.
2. Elements of the representation theory of PSL (2, R)
   - PSL (2, R) group of isometries of the Poincaré plane.
   - Models of unitary irreducible representations of PSL (2, R)
   - We admit that these models exhaust the unitary dual of PSL (2, R)
   - C^∞ vectors and Harish-Chandra modules (infinitesimal point of view)
   - Splitting of $L^2(G/\Gamma)$ into unitary irreducible representations
   - Geometric interpretation of the decomposition of $L^2(G/\Gamma)$ into irreducible modules and spectral gap
   - Sobolev vectors in $L^2(G/\Gamma)$

Lecture 3 (90 min)

3. Invariant Distributions and cohomological equation for horocycle flow.
   - Sobolev estimate of solutions.
4. Renormalization and ergodic averages.
   - Gottschalk-Hedlund revisited: estimate for the component orthogonal to the invariant distributions for a segment of horocycle orbit
   - Action of the geodesic flow on the space of invariant distributions for horocycle flow
   - Renormalization for compact surfaces.

Lecture 4 (90 min)

5. Elements of Kirillov theory,
   - Nilpotent groups
   - Coadjoint orbits, polarizing subalgebras,
   - Unitary dual of nilpotent groups: Kirillov's classification.
   - Example: Heisenberg group
   - C^∞ vectors and Sobolev vectors for irreducible representations of nilpotent groups.

Lecture 5 sand 6 (180 min)

6. Cohomological equation cohomological for nilpotent groups
   - Sobolev estimate of solutions.
7. Renormalization and ergodic averages for Heisenberg's group.
   - Automorphisms of Heisenberg's group and moduli of lattices
   - Bundle of invariant distributions over moduli space
   - The Renormalization dynamics.
8. Related works: $SL(2, C)$,... Higher order cohomology. Difficulties and perspectives.

Prerequisites

Basic definitions on Lie algebra and Lie Groups (any book on the subject, for exemple Helgason); Spectral theory of unbounded self-adjoint operators (Reed-Simon: (Functional analysis) or Riesz-Nagy(Functional analysis)) Distributions (Reed-Simon: (Functional analysis) suffices, but see also Hormander (Linear differential operators)). Elliptic regularity and Sobolev spaces (There are many books on the subject: Warner (Foundation of differential geometry) has a nice chapter on it; standard references are Gilbarg-Trudinger (Elliptic...) and, of course, Hörmander (Linear differential operators)).

Tentative division of lectures:

- Lectures 1-3: L. Flaminio
- Lectures 4-6: G. Forni