**MASS-07; GEOMETRY**

FALL 2007

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HOMEWORK # 10

November 9, 2007

Due on Friday, November 16

*CONTROL PROBLEMS: You should do these problems independently without consulting other students.*

**39.** Classify orientation reversing isometries of the hyperbolic plane. Use linear algebra first and then give a geometric interpretation following as closely as possible similarity with the isometries of Euclidean plane.

**40.** Calculate the hyperbolic radius and the hyperbolic center of the circle in $H^2$:

$$\|z - 2i - 1\|^2 = 9/4.$$

REGULAR PROBLEMS:

**41.** Assume you know that the sum of the angles of any geodesic triangle in the hyperbolic plane is less that $\pi$. Prove that any two geodesic triangles with pairwise equal angles are isometric.

**42.** Prove that any orientation preserving isometry of $H^2$ is the product of two reflections and any orientation reversing isometry is the product of three reflections.

**43.** Prove that the group of orientation preserving isometries of $H^2$ in the unit disc model is the group of all fractional linear transformations of the form

$$z \to \frac{az + \bar{c}}{cz + \bar{a}}$$

where $a, c \in \mathbb{C}$, $a\bar{a} - c\bar{c} = 1$.

OPTIONAL PROBLEM (deadline November 30 ):

**O6.** Recall that a subgroup $H$ of a group $G$ is *normal* if for any $h \in H$ and $g \in G$ the conjugate $g^{-1}hg \in H$. Recall that translations form a normal subgroup in the group of isometries of Euclidean plane.

Prove that the group of isometries of the hyperbolic plane has no non-trivial normal subgroups, i.e. the only normal subgroups are the whole group and identity.

*Hint:* Use classification of conjugacy classes in $SL(2, \mathbb{R})$.