MASS-07; GEOMETRY

FALL 2007

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HOMEWORK # 12 (the last)

November 30, 2007

Due on Wednesday, December 5

51. Let $\gamma : S^1 \to \mathbb{R}$ be a closed curve and $f : S^1 \to S^1$ a continuous map. Let $\gamma_f(t) = \gamma(f(t))$. Prove that

$$\mathrm{ind}_x \gamma_f = \deg f \cdot \mathrm{ind}_x \gamma.$$

52. Let $\gamma : S^1 \to \mathbb{R}$ be a smooth regular closed curve with one transversal (non-tangential) self-intersection, i.e. the curve intersects itself under a non-zero angle. Prove that the complement to γ consists of three connected components and list (with a proof) all possibilities for indices of the points in those components with respect to γ in those components.

53. Construct on every closed compact surface a smooth vector-field with one zero and show by direct calculation that index of this zero is equal to the Euler characteristic.

Hint: You may use standard models to construct non-smooth vector fields first and make them smooth by multiplying to a scalar function positive outside on this this point and decreasing fast toward it.