MASS-07; GEOMETRY

FALL 2007

A.Katok

HOMEWORK # 4

September 14, 2007

Due on Friday, September 21

CONTROL PROBLEMS: You should do these problems independently without consulting other students.

13. Consider the space obtained from the torus $\mathbb{T}^2 = \mathbb{R}^2/\mathbb{Z}^2$ by identifying every point x with $-x$. Prove that it is homeomorphic to the sphere.

14. Calculate Euler characteristic of the Klein bottle.

REGULAR PROBLEMS:

15. A regular polyhedron in $\mathbb{R}^3$ is characterized by its Schläfly symbol (p, q) where p is the number of sides of a face and q is the number of edges at each vertex. Prove that there are only five possible Schläfly symbols corresponding to five known Platonic solids.

16. Find the minimal possible number of two-simplices for a triangulation of the projective plane.

17. Define the notion of triangulation for a surface with boundary, prove invariance of Euler characteristic and calculate Euler characteristic of the sphere with m holes.

OPTIONAL PROBLEM (deadline October 1)

O2. Prove that there are only five regular polyhedra up to a similarity transformation (isometry and homotety).