

# MASS-07; GEOMETRY

FALL 2007

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HOMEWORK # 5

September 21, 2007

Due on Friday, September 28

*CONTROL PROBLEMS: You should do these problems independently without consulting other students.*

**18.** Consider all surfaces which can be obtained by identifying pairs of sides in a hexagon. Divide those into groups of mutually homeomorphic ones and prove that surfaces from different groups are not homeomorphic

**19.** Prove equivalence of two definitions of orientability for a surface with a triangulation: (i) all triangles can be oriented in a coherent way, and (ii) one can choose a positive direction of rotation at every point which changes continuously.

Prove that orientability does not depend on the choice of triangulation.

REGULAR PROBLEMS:

**20.** Prove that attaching a handle to a non-orientable surface and attaching two Mobius caps to the same surface produce homeomorphic surfaces.

**21.** Find a necessary and sufficient condition on  $k$  and  $l$  so that there is a covering map from the sphere with  $k$  handles onto the sphere with  $l$  handles.

**22.** Prove that for every  $m \geq 1$  both the regular  $4m$ -gon and the regular  $4m+2$ -gon with pairs of opposite sides identified by translations are homeomorphic to the sphere with  $m$  handles.

*You may use the standard polygonal model for the sphere with handles but cannot refer to the general classification theorem: you have to produce an explicit construction*