

# MASS-07; GEOMETRY

FALL 2007

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HOMEWORK # 6

October 7, 2007

Due on Friday, October 12

*CONTROL PROBLEMS: You should do these problems independently without consulting other students.*

**23.** Construct a smooth atlas for the standard flat torus which contains only three charts.

**24.** Prove that the standard flat torus is diffeomorphic to the standard torus of revolution.

REGULAR PROBLEMS:

**25.** Calculate  $\mathbb{Z}$ -homology of the sphere with  $m$  Mobius caps.

**26.** Prove that for a compact surface in  $\mathbb{R}^3$  given by the equation  $F(x, y, z) = 0$  where  $F$  is a differentiable function and 0 its regular value one can obtain a smooth atlas by choosing a certain finite set of points and projecting balls of a sufficiently small radius in the tangent spaces of those points to the surface.

**27.** Let  $F$  be a differentiable convex function in  $\mathbb{R}^3$ , i.e.  $F(\mathbf{x} + \mathbf{y}) \geq F(\mathbf{x}) + F(\mathbf{y})$ . If  $c$  is a regular value of  $F$  and the surface  $F = c$  is nonempty and compact then this surface is diffeomorphic to the standard sphere  $x^2 + y^2 + z^2 = 1$ .