

MASS-07; GEOMETRY

FALL 2007

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HOMEWORK # 7

October 12, 2007

Due on Friday October 19

CONTROL PROBLEM: You should do this problem independently without consulting other students.

28. Prove that stereographic projections from north and south poles form an atlas on the sphere and that this atlas is compatible with the one with six charts obtained by the projections to coordinate planes.

REGULAR PROBLEMS:

29. Prove that if in an atlas on a surface (compact or not) the transfer maps between any two charts have positive determinant at any point of intersection, then the surface is orientable.

30. Using the fact that the orientable surface of genus m can be obtained from two copies of the sphere with $m+1$ (round) holes pairwise identified along their boundaries construct a smooth structure on such a surface.

OPTIONAL PROBLEM (deadline October 28)

O3. Construct a *complex diffeomorphism*, i.e. a function $z \mapsto F(z)$ differentiable with its inverse as a function of one complex variable, between the unit disc in \mathbb{C} , $|z| < 1$ and the unit square $0 < \operatorname{Re} z < 1$, $0 < \operatorname{Im} z < 1$.