

MASS-07; GEOMETRY

FALL 2007

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HOMEWORK # 8

October 26, 2007

Due on Friday, November 2

CONTROL PROBLEM: You should do this problem independently without consulting other students.

31. Express the Riemannian metric on the round sphere in the following coordinate systems:

- (1) geographic coordinates;
- (2) polar coordinates in the planes of stereographic projections;
- (3) polar coordinates in the planes of coordinate projections.

REGULAR PROBLEMS:

32. An *integral curve* of a vector field X is a smooth curve such that X is tangent to the curve at any point of the curve.

Construct a smooth vector field on the torus without zeroes and without closed integral curves.

Hint: Look at vector fields in the plane which can be projected to the torus.

33. Consider a vector field X and a simple closed curve γ which surrounds a disc D on the surface such there are only isolated zeroes of the vector field inside the disc and no zeroes on γ . The index of X along γ is defined as before. Prove that this index is equal to the sum of indices of zeroes inside D .

34. Construct an example of a vector field on the sphere with a single zero and calculate index of this zero.

Hint: You may start with a “natural” vector field and try to deform it to glue its zeroes together.

35. Consider the regular octagon with pairs of opposite sides identified and with smooth structure defined in section d of Lecture 19 (see figure 7 on p. 83).

Define a smooth Riemannian metric on this surface in such a way that *angles* between tangent vectors at any point other than the vertex are equal to Euclidean angles.

Hint: Remember that we constructed smooth structure using conformal transformations.

OPTIONAL PROBLEMS (deadline November 12):

O4. Prove characterization of the degree for a smooth circle map as the number of “positive” pre-images minus the number of “negative” pre-images for any non-critical value.

WARNING: you must show that non-critical values exist!

O5. It is true that index of an isolated zero of a smooth vector field can take any integer value?