MASS-09; ALGEBRA

FALL 2009

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HOMEWORK # 1

Due on Wednesday September 9

1. Prove that any homomorphic image of the group $\mathbb{Z}$ is a cyclic group.

2. (1) Prove that the group of all complex numbers of absolute value one by multiplication is isomorphic to the factor-group $\mathbb{R}/\mathbb{Z}$.

(2) Prove that the group of non-zero all complex numbers by multiplication is isomorphic to the group $\mathbb{R}^2/G$, where G is the group of vectors of the form $(n, 0)$, $n \in \mathbb{Z}$.

3. Consider S_3, the group of all permutations of three elements. It has three subgroups of order 2. Describe explicitly right and left cosets for each of these subgroups.

4. Find all possible orders of various elements in the symmetric groups S_6 and S_7.

5. Find the number of *cyclic* permutations of n elements.

6. (1) Find the number of different elements of order 2 in the symmetric group S_n.

(2) Find the number of different elements of order 2 in the alternating group A_n, or equivalently, the number of different even involutions of n elements.

7. Consider the group $G = \mathbb{Z}_/2\mathbb{Z} \times \mathbb{Z}/2\mathbb{Z} \times \mathbb{Z}/2\mathbb{Z}$

(1) Prove that G is the only group of order 8 all of whose element have order 2.
(2) Embed G into symmetric group S_6, i.e. find a subgroup of S_6 isomorphic to G
(3) Prove that G cannot be embedded into S_5, i.e the latter group does not contain a subgroup isomorphic to G.