MASS-09; ALGEBRA

FALL 2009

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HOMEWORK # 10

Due on WEDNESDAY, November 18

47. Find a closed formula for the growth function for the group $\mathbb{Z}^k$ with standard generators.

48. Prove that any discrete group of isometries of Euclidean space of any dimension has polynomial growth.

49. Consider the group of transformations of the line generated by $T : x \to x+1$ and $H : x \to 2x$.

- Find generating relations.
- Does this group have exponential or sub-exponential growth?

50. Let N_3 be the group generated by matrices

$$n_1 = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix},\ c = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \text{ and } n_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}.$$

- Find the growth function for N_3 with respect to (n_1, n_2, c).
- Construct the Cayley graph for the group N_3 with generators n_1, n_2, c.

51. Recall that $G * H$ is the free product of groups G and H.

- Prove that $G * H$ has exponential growth if at least one of the groups has more than two elements.
- Find growth function for $\mathbb{Z}/2\mathbb{Z} * \mathbb{Z}/2\mathbb{Z}$.