

MASS-09; ALGEBRA

FALL 2009

A.Katok

HOMEWORK # 11 (the last)

Due on MONDAY, NOVEMBER 30

52. Prove that the group $Out(F_k)$, $k \geq 2$ is not free and does not have a free subgroup of finite index

53. Prove that any subgroup of a finitely generated nilpotent group is finitely generated

54. Prove that the solvable group from Problem 49 contains an abelian subgroup that is not finitely generated.

55. Prove that the Group Γ_3 is not a semi-direct product of \mathbb{Z} and \mathbb{Z}^2 .

56. Suppose a group G does not have a subgroup of index two. Prove that the only semi-direct product of G with \mathbb{Z} is direct.

57. Prove that if groups of polynomial growth are quasi-isometric then the (upper) degree of growth is the same.

58. Give an example of two groups that are quasi-isometric but not commensurable.