MASS-09; ALGEBRA

FALL 2009

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HOMEWORK # 2

Due on Wednesday, September 16

8. Prove that the number of different conjugacy classes in the symmetric group S_n is equal to the number of different ways the number n can be represented as the sum of (not necessarily different) positive integers.

9. Prove that any orientation preserving (even, direct) isometry of $\mathbb{R}^3$ that has a fixed point, has infinitely many fixed points.

10. Find all isometries in $Iso(\mathbb{R}^2)$ that commute with a given translation.

11. A group of isometries (or any transformations of $\mathbb{R}^n$) is called *discrete* if the set of images of any point has no accumulation points.

(1) Prove that any discrete group $G \subset Iso(\mathbb{R}^2)$ contains a translation subgroup of finite index.
Hint: *Assume the opposite and consider the subgroup* $G \cap Iso^+(\mathbb{R}^2)$.

(2) Give an example of an infinite discrete group $G \subset Iso(\mathbb{R}^3)$ that contains no translations (except for the identity).

12.

(1) Describe the full symmetry group of the integer lattice $\mathbb{Z}^2$ in the plane.

(2) Give an example of a "nice" set in $\mathbb{R}^2$ whose full symmetry group is the group $\mathbb{Z}^2$ of translations by vectors with integer coordinates.