MASS-09; ALGEBRA

FALL 2009

A.Katok

HOMEWORK # 5

Due on FRIDAY, OCTOBER 9 (after the midterms)

22. Consider the group G of isometries of $\mathbb{R}^2$ generated by reflections in the sides of a triangle T.

(1) Prove that G is discrete if and only if T belongs to one of the four types: (i) regular; isosceles right; (iii) right with an angle $\pi/6$; isosceles with an angle $2\pi/3$;
(2) In cases (i),(ii),(iii),(iv) as above, describe the finite index subgroups of translations in G.[1]

23. Prove that an upper-triangular matrix is normal if and only if it is diagonal.
Hint: Look at diagonal elements. Induction in dimension may help.

24. Let $SU(2)$ be the group of complex unitary 2×2 matrices with determinant one. Prove that there is a continuous bijective map from the group $SU(2)$ onto the sphere $S^3 \subset \mathbb{R}^4$.

25.(1) Prove that the group $\mathrm{Aff}\,(\mathbb{R}^2)$ is isomorphic to the group of projective transformations of the projective plane preserving a fixed line.

(2) Find a similar statement in lower dimension.

[1]there was a minor error in the original formulation: (iv) was omitted.