MASS-09; ALGEBRA

FALL 2009

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HOMEWORK # 6

Due on WEDNESDAY, October 21

26. Describe conjugacy classes in the group $SL(2,\mathbb{C})$. Give geometric interpretation as conformal maps of the Riemann sphere.

27. Find necessary and sufficient conditions for the anti-fractional linear transformation $z \to \frac{a\bar{z}+b}{c\bar{z}+d}$, $a,b,c,d \in \mathbb{C}$ of the Riemann sphere $\mathbb{C}\cup\{\infty\}$ to be

(i) inversion wrt. a circle,

(ii) reflection in a line.

Find the circle of inversion and the axis of reflection.

28. Let $SL(2,\mathbb{Z})$ be the group of integer matrices with determinant 1. Let $A = \begin{pmatrix} 1 & 2 \\ 1 & 3 \end{pmatrix}$, $B = \begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix}$. Prove that A and B are not conjugate in $SL(2,\mathbb{Z})$.

29. Consider the stationary subgroup S_z of $Isom(\mathbb{H}^2)$ (i.e. fractional linear and anti-fractional linear transformations with real coefficients) fixing the point $z \in \mathbb{H}^2$.

(1) Prove that S_z is generated by inversions in circles centered on the real line and passing through z.
(2) Interpret the orientation preserving subgroup of S_z as hyperbolic rotations around z.

30. A *general torus* T is the factor–group $\mathbb{R}^2/L$, where L is the lattice generated by two linearly independent vectors vectors (a,b), and (c,d). The (Euclidean) distance in T is defined as the minimum of Euclidean distance between the elements of corresponding cosets.

(1) Prove that every isometry of T comes from $\mathbb{R}^2$.
(2) Prove that T always has isometries other than translations.
(3) Find a necessary and sufficient condition for T to possess an orientation reversing isometry.

31. Find the center of the group of 3×3 matrices of the form $\begin{pmatrix} 1 & m & k \\ 0 & 1 & n \\ 0 & 0 & 1 \end{pmatrix}$ where m, n and k are integers.