MASS-09; ALGEBRA

FALL 2009

A.Katok

HOMEWORK # 7

Due on WEDNESDAY, October 28

32. Find $\exp A$ where $A = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$.

33. Matrix A with real entries is *skew-symmetric* if $A^t = -A$. Prove that skew-symmetric $n \times n$ matrices form a Lie algebra.

34. Complex matrix A is *skew-Hermitian* if $A^t = -\bar{A}$. Prove that

(1) if A is skew-symmetric then $\exp A$ is orthogonal;
(2) if A is skew-Hermitian, then $\exp A$ is unitary.

35. Prove that the fundamental group $\pi_1(SO(3)) = \mathbb{Z}/2\mathbb{Z}$

36. *Mobius strip* is a surface in $\mathbb{R}^3$ obtained by moving a segment of length $1/2$ with the midpoint on the unit circle in the (x, y) plane keeping it perpendicular to that circle and rotating uniformly around the midpoint in such a way that the total angle of rotation after the midpoint returns to the original position, is equal to π.

Find the fundamental group of the Mobius strip.