# MASS-09; ALGEBRA

FALL 2009

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HOMEWORK # 8

Due on WEDNESDAY, November 4

**37.** Prove that $\pi_1(SL(2,R)) = \mathbb{Z}$.

**38.** Consider the subgroup $G$ of $Isom(R^2)$ generated by the translation $T(x,y) = (x+1,y)$ and the glide reflection $G(x,y) = (-x,y+1)$.

(1) Prove that $G$ acts on $\mathbb{R}^2$ freely and in the discrete fashion.
(2) Prove that $G$ is non-abeloian and has an abelian subgroup of index two.
(3) Let $K = R^2/G$ be the factor-space, Show that there is a two-to-one covering map $c : \mathbb{T}^2 \to K$.

**39.** Let $c : X \to S^1$ be a covering map. Prove that $X$ is either the line or the circle.

**40.** Find subgroups of $F_2$ isomorphic to $F_n$, $n = 1, 2, 3, \ldots, \infty$

**41.** Show that $F_2$ has infinitely many non-conjugate subgroups of finite index.