

MASS-09; ALGEBRA

FALL 2009

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HOMEWORK # 9

Due on WEDNESDAY, November 11

42. Find generators and generating relations in the Platonic bodies rotation groups A_4 , S_4 and A_5 .

43. Find a subgroup of isometries of hyperbolic plane isomorphic to F_∞ , the free group with infinitely many generators. Describe its generators explicitly.

44.

(1) Prove that matrices $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ and $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ generate the group $SL(2, \mathbb{Z})$ and find generating relations.

(2) Find two matrices of finite order that generate the group $SL(2, \mathbb{Z})$ and find generating relations.

(3) Find the group generated by matrices $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ and $\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$ and find generating relations.

Hint: Look at the group as a subgroup of $SL(2, \mathbb{Z})$.

45. Find infinitely many non-conjugate normal subgroups of finite index in F_3 .

46. Prove that the group of similarity transformations in \mathbb{R}^2 does not contain a free subgroup with $n \geq 2$ generators.