MASS-09; ALGEBRA

FALL 2009

A.Katok

MIDTERM EXAMINATION

Tuesday, October 6 2009

SECTION 1. PROBLEMS

Provide complete solutions with proofs of any two *of the problems.*

1.1. Prove that the only non-abelian group of order 10 is the dihedral group D_5.

1.2. Find matrix representations for the following groups:

(1) Orientation preserving isometries in $\mathbb{R}^3$ preserving a given plane.
(2) Affine maps in $\mathbb{R}^3$ preserving a given plane.

1.3. Let T be factor–group $\mathbb{R}^2/L$, where L is the lattice generated by vectors $(1,0)$, and $(1/2, \sqrt{3}/2)$ (T is another torus). The (Euclidean) distance in T is defined as the minimum of Euclidean distance between the elements of corresponding cosets.

Describe the group of isometries of T with Euclidean distance.

1.4. Prove that if an orientation preserving isometry in $\mathbb{R}^n$ has a non-fixed point of period two, it has infinitely many such points.

SECTION 2. THEORETICAL QUESTIONS

2.1. List all conjugacy classes in the group $Isom(\mathbb{R}^3)$.

2.2. List all finite groups of isometries (not only rotation groups) in $\mathbb{R}^3$.

SECTION 3. QUESTIONS

Give complete answers. Any explanations/proofs are optional.

3.1. How many different elements of orders 3, 4 and 5 are in the symmetric group S_9? How many of those permutations are even?

3.2. Name two non-isomorphic non-abelian groups of order 12.

3.3. Consider the group G generated by reflections in the sides of the regular hexagon. List all normal subgroups of G.