MASS-11; ANALYSIS

FALL 2011

A.Katok

HOMEWORK # 3

Due on Wednesday September 14

11. Let for $\mathbf{x} = (x_1, \dots, x_n)$

(1) Find the maximal number C_n such that

$$\|\mathbf{x}\|_2 \geq C_n \|\mathbf{x}\|_1.$$

(2) Find the minimal number C_n such that

$$\|\mathbf{x}\|_2 \leq C_n \|\mathbf{x}\|_\infty.$$

12. Give a geometric interpretation of the results of the previous problem for $n = 3$. Provide a drawing.

13. A norm in $\mathbb{R}^n$ is called *strictly convex* if $\|x + y\| = \|x\| + \|y\|$ implies that $y = \alpha x$ for an $\alpha \in \mathbb{R}$.

Prove that the norm is strictly convex if and only if the unit "sphere" $B = \{x : \|x\| = 1\}$ does not contain a line segment.

14. Given a basis $\mathbf{x}_1, \dots, \mathbf{x}_n$ in $\mathbb{R}^n$ find the number of different orthonormal bases $\mathbf{y}_1, \dots, \mathbf{y}_n$ such that for $k = 1, \dots, n$ the vector $\mathbf{y}_k$ is a linear combination of $\mathbf{x}_1, \dots, \mathbf{x}_k$.

EXTRA CREDIT PROBLEM (Due September 28)

2*.Define the sum $A + B$ of two sets in $\mathbb{R}^n$ as the set of all sums $x + y$ where $x \in A$ and $y \in B$. Similarly define αA for $\alpha \in \mathbb{R}$.

Prove that for convex bounded sets $A_1, \dots, A_k$ in $\mathbb{R}^2$ the area of the set $\alpha_1 A_1 + \dots + \alpha_k A_k$ is a quadratic form in variables $\alpha_1, \dots, \alpha_k$.

If you want a real challenge, try to generalize this problem to arbitrary dimension.