

MASS-11; ANALYSIS

FALL 2011

A.Katok

HOMEWORK # 4

Due on Wednesday September 21

15. Prove Holder inequality using the method of Lagrange multipliers. Deduce using the same method that the norm dual to $\|\cdot\|_p$ is $\|\cdot\|_q$ where $q = \frac{p}{p-1}$.

16. Octacube, on which professor Adrian Ocneanu lectured on Tuesday, is a four-dimensional polyhedron with 24 vertices:

$\{\pm 1, \pm 1, \pm 1, \pm 1\}$, $\{\pm 2, 0, 0, 0\}$, $\{0, \pm 2, 0, 0\}$, $\{0, 0, \pm 2, 0\}$, $\{0, 0, 0, \pm 2\}$.

Write down explicitly the norm in \mathbb{R}^4 for which the octacube is the unit ball.

17. Suppose that a norm in \mathbb{R}^n is a function differentiable outside of the origin. Prove that the dual norm is strictly convex, i.e. in that norm the triangle inequality is strict unless the vectors are positive scalar multiples of each other.

18. Prove that the dual space to the space $C([0, 1])$ is not separable.

19. Find the dual space to the space l^1 of infinite absolutely summable sequences with the norm $\|x\| = \sum_{n=1}^{\infty} |x_n|$.