MASS-11; ANALYSIS

FALL 2011

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HOMEWORK # 6

Due on Wednesday October 5

23. Let B be the closed unit ball in the space ℓ^1 of absolutely summable sequences.

(1) Prove that not every point of B can be represented as
$$\sum_{i=1}^{n} s_i x_i$$
where x_i, $i = 1, \dots, n$ are extreme points, $0 \leq s_i \leq 1$, $\sum_{i=1}^{n} s_i = 1$.

(2) Prove that every point of B is the limit of points of above form.

24. Consider a norm in $\mathbb{R}^n$ such that the unit ball B is a convex polyhedron. Construct a natural correspondence between k-dimensional faces of B and $n-k$-dimensional faces of the unit ball B' in the dual norm.

25. Consider a norm in $\mathbb{R}^4$ such that the unit ball B is the octacube. Describe the dual norm in $\mathbb{R}^4$. Find a coordinate system in which the unit ball in the dual space is the octacube.

26. Prove that if in a family of of convex subsets in $\mathbb{R}^n$ every $n+1$ sets in the family have a non-empty intersection, then all sets from the family have a non-empty intersection.

Hint: Show that it is enough to consider finite families. After that use induction in the number of sets and in the dimension.

27. Find a two-dimensional subspace L of the space $C([0,1])$ such that the restriction of the norm to this space makes it isometric to the plane $\mathbb{R}^2$ with Euclidean norm.

EXTRA CREDIT PROBLEMS (Due October 19)

3*. Prove that there does not exist a Banach space whose dual is isometric to ℓ^1.

4*. Prove that $\mathbb{R}^2$ with any norm can be isometrically embedded into $C([0,1])$, i.e. as in the problem 27 there exists a a two-dimensional subspace L of the space $C([0,1])$ such that the restriction of the norm to this space makes it isometric to the plane $\mathbb{R}^2$ with the given norm.

5*. Let B be the unit ball in the dual space to a separable Banach space. Prove that every element of B is the limit os elements of the form

$$\sum_{i=1}^{n} s_i x_i$$

where x_i, $i = 1, \ldots, n$ are extreme points of B, $0 \leq s_i \leq 1$, $\sum_{i=1}^{n} s_i = 1$.