MASS-11; ANALYSIS

FALL 2011

A.Katok

HOMEWORK # 9

Due on Wednesday November 9

39. Prove that the space of continuous functions $C([0,1])$ is of the first Baire category (union of countably many nowhere dense sets) in its completion with respect to any L^p, $p \geq 1$ norm:

$$\|f\| = (\int |f(x)|^p dx)^{1/p}.$$

40. Let X be a compact metric space. Prove that X is homeomorphic to a closed subset of the Hilbert cube (the product of countably may copies of the unit interval).
Hint: Consider the space of continuous functions on X.

41. Let $\mathbf{a} = a_1, a_2, \dots$ be a sequence of positive numbers. Consider the following subset of the Hilbert space l^2:

$$C_{\mathbf{a}} = \{|x_n| \leq a_n, \|x\|_2 \leq 1\}.$$

Give a necessary and sufficient condition on the sequence $\mathbf{a}$ such that the set $C_{\mathbf{a}}$ is compact.

42. Consider the space of symmetric polynomials in n variables defined on a compact subset K of $\mathbb{R}^n$. Describe with proofs the closure of this space in uniform topology.

43. Prove that Fourier coefficients f_n of a continuous function f on the circle

$$f_n = \int_0^1 f(x) \exp(2\pi i n x) dx$$

converge to zero as $n \to \infty$.