REU SUMMER 2007; July 16–27 2007;
Ten two-hour lectures, one on every weekday.

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GROUPS AND THIER CONNECTIONS TO GEOMETRY

1. Different ways groups appear:
a. Groups by addition or multiplication in arithmetic and algebra
b. Symmetries in geometry
c. Permutation groups
d. Matrix (or linear transformations) groups
e. Common framework for b. c. and d. : Transformation groups

2. Finite groups.
Multiplication table. Cyclic, dihedral, symmetric and alternating groups. Generators and relations Basic properties: order of elements, normal subgroups, factors Classification of groups with small number of elements: (up to 11 in class; up to 24 as a project)

3. Infinite groups determined by generators and relations.
Examples: Free abelian and non-abelian groups, free products, some nilpotent groups. Finitely generated and finitely presented groups.

4. Geometric objects associated with groups:
a. Cayley graph, examples.
b. Discrete groups as lattices. Examples: euclidean space, hyperbolic plane.
c. Various types of boundaries.

5. Growth in groups.
Invariance of exponential growth under the choice of generators. Growth invariants. Examples.

6. Groups of polynomial growth.
Nilpotent groups and their growth. Examples. Principle of commensurability. Classification of groups with slow growth (up to commensurability).

7. Gromov theorem.
Formulation, discussion, outline of proof.

8. Problem of intermediate growth.

Grigorchuk groups.

9. Some easily formulated open problems.