MATH 527: GEOMETRY/TOPOLOGY I

FALL 2006

A.Katok

HOMEWORK # 10

Surfaces I: triangulations, functions, Möbius caps

Due on Monday December 4

47. Prove that there exists a triangulation of the projective plane with any given number $N > 4$ of vertices.

48. Prove that minimal number of vertices in a triangulation of the torus is seven.

Hint: Use Euler theorem.

49. Prove that for any triangulation $\mathcal{T}$ of a surface there exists a smooth function whose local maxima are vertices of $\mathcal{T}$ and which has exaclty one saddle on each edge of $\mathcal{T}$, exaclty one local minimum inside each face of $\mathcal{T}$ and no more critical points.

50. Construct a smooth function of the torus with three critical points.

In the next two problems you must describe the homeomorphisms explicitly and not refer to the general theorem about classification of surfaces.

51. Given a surface M attaching a *Möbius cap* consists of deleting a small disk and identifying the resulting boundary circle with the boundary of a Möbius strip.

Prove that the sphere with two Möbius caps attached is homeomorphic to the Klein bottle.

52. Prove that sphere with three Möbius caps attached is homeomorphic to the torus with a Möbius cap attached.