

**MATH 527: GEOMETRY/TOPOLOGY I**

FALL 2006

**A.Katok**

HOMEWORK # 11

Surfaces II: Classification, fundamental group, covering spaces

Due on Monday December 11

*You cannot use classification of surfaces for the next two problems*

**53.** Prove that attaching a handle decreases Euler characteristic of a compact surface (with or without boundary) by two.

**54.** Prove that attaching a Möbius cap decreases Euler characteristic of a compact surface (with or without boundary) by one.

**55.** Prove that the only compact surfaces from the standard list (spheres with handles, Möbius caps, and holes) which have abelian fundamental group are the sphere, the closed cylinder, the closed Möbius strip, the projective plane, and the torus.

**57.** Prove that any compact covering space for the torus is another torus.

**58.** Prove that the orientable surface of genus  $m$  is a covering space for an orientable surface of genus  $n \geq 2$  if and only if  $m > n$  and  $n - 1$  divides  $m - 1$ .