# MATH 527: GEOMETRY/TOPOLOGY I

## FALL 2006

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HOMEWORK # 4; October 2, 2006

Manifolds, graphs, beginning homotopy

Due on Monday October 9

**16.** Consider the torus $\mathbb{T}^2 = \mathbb{R}^2/\mathbb{Z}^2$ and let $S$ be the quotient space obtained by identifying orbits of the map $I : x \mapsto -x$. Prove that $S$ is homeomorphic to the sphere $\mathbb{S}^2$.

**17.** Ex 2.3.1.

**18.** Ex. 2.12.1.

**19.** Ex 2.12.2.

**20.** Ex 2.12.4.

## "'Extra credit" problems
*You may submit solutions until October 27.*

**E4.** Consider regular $2n$-gon and identify pairs of opposite side by the corresponding parallel translations. Prove that the identification space is a topological manifold. Prove that the manifolds obtained by this construction from the $4n$-gon and and $4n + 2$-gon are homeomorphic.

**E5.** Prove that the manifold of the previous exercise is homeomorphic to the surface of the sphere to which $n$ "handles" are attached, or, equivalently, to the surface of $n$ tori joint into a "chain" (Figure 1.8.1 illustrates this for $n = 1$ and $n = 3$.