MATH 527: GEOMETRY/TOPOLOGY I

FALL 2006

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HOMEWORK # 5: October 9, 2006

Fundamental group, covering spaces

Due on Monday October 16

21. Prove that for any path connected topological space X we have $\pi_1(\text{Cone(X)}) = 0$.

22. Consider the following map f of the torus $\mathbb{T}^2$ into itself:

$$f(x, y) = (x + \sin 2\pi y, 2y + x + 2\cos 2\pi x) \quad (\mod 1).$$

Describe the induced homomorphism f_* of the fundamental group.

23. Let $X = \mathbb{R}^2 \setminus \mathbb{Q}^2$. Prove that $\pi_1(X)$ is uncountable.

24. Let X be the quotient space of the disjoint union of $\mathbb{S}^1$ and $\mathbb{S}^2$ with a pair of points $x \in S^1$ and $y \in S^2$ identified. Calculate $\pi_1(X)$.

25. Describe two-fold coverings of

(1) the (open) Möbius strip by the open cylinder $\mathbb{S}^1 \times \mathbb{R}$;
(2) the Klein bottle by the torus $\mathbb{T}^2$.

26. Prove that the real projective space $\mathbb{R}P(n)$ is not simply connected.

Hint: Use the fact that $\mathbb{R}P(n)$ is the sphere $\mathbb{S}^n$ with diametrically opposed points identified.