

MATH 527: GEOMETRY/TOPOLOGY I

FALL 2006

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HOMEWORK # 6; October 23 2006

Hopf fibration, Cantor sets, differentiable manifolds

Due on Monday October 30

26. For any finite cyclic group C there exists a compact connected three-dimensional manifold whose fundamental group is isomorphic to C .

Hint: Use the Hopf fibration.

27. For any abelian finitely generated group A there exists a compact manifold whose fundamental group is isomorphic to A .

Hint: Use the fact that any finitely generated abelian group is the direct product of cyclic groups (finite and infinite).

28. Prove that complex projective space $\mathbb{C}P(n)$ is simply connected for every n .

29. Introduce a metric d on the Cantor set C (generating the Cantor set topology) such that (C, d) cannot be isometrically embedded to \mathbb{R}^n for any n .

30. Construct a smooth atlas of the projective space $\mathbb{R}P(3)$ with as few charts as possible.

“Extra credit” problems

You may submit solutions until November 14.

E6. Introduce a metric d on the Cantor set C such that (C, d) is not Lipschitz equivalent to a subset of \mathbb{R}^n for any n .

E7. Prove that for any finite graph G , $\pi_n(G) = 0$ for any $n \geq 2$.