MATH 527: GEOMETRY/TOPOLOGY I

FALL 2006

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HOMEWORK # 7; October 30, 2006

Differentiable manifolds; diffeomorphisms, submanifolds

Due on Monday, November 6

31. Construct an explicit diffeomorphism between $\mathbb{R}^n$ and the open unit ball B^n.

32. Prove that any convex open set in $\mathbb{R}^n$ is diffeomorphic to $\mathbb{R}^n$.

33. Prove that the following three smooth structures on the torus $\mathbb{T}^2$ are equivalent, i.e. the torus provided with any of these structure is diffeomorphic to the one provided with another:

- $\mathbb{T}^2 = \mathbb{S}^1 \times \mathbb{S}^1$ with the product structure;
- $\mathbb{T}^2 = \mathbb{R}^2/\mathbb{Z}^2$ with the factor-structure;
- The embedded torus of revolution in $\mathbb{R}^3$
  $$\mathbb{T}^2 = \left\{(x,y,z) \in \mathbb{R}^3 : \left(\sqrt{x^2+y^2} - 2\right)^2 + z^2 = 1\right\}$$
  with the submanifold structure.

34. Prove that the n-dimensional torus in $\mathbb{R}^{2n}$:

$$x_{2k-1}^2 + x_{2k}^2 = \frac{1}{n}, \quad k = 1, \dots, n$$

is a smooth submanifold of the $(2n-1)$-dimensional sphere

$$\sum_{i=1}^{2n} x_i^2 = 1.$$

35. Prove that the upper half of the cone

$$x^2 + y^2 = z^2, \;\; z \geq 0$$

is not a submanifold of $\mathbb{R}^3$, while the punctured one

$$x^2 + y^2 = z^2, \;\; z > 0$$

is a submanifold of $\mathbb{R}^3$.