

**MATH 527: GEOMETRY/TOPOLOGY I**

FALL 2006

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HOMEWORK # 8; November 10, 2006

Tangent bundles, examples of Lie groups, orientability

Due on Monday November 20

- 36.** Prove that the tangent bundle to the three-dimensional sphere  $\mathbb{S}^3$  is diffeomorphic to the direct product  $\mathbb{S}^3 \times \mathbb{R}^3$ .
- 37.** Find a natural smooth group structure on the sphere  $\mathbb{S}^3$ .
- 38.** Prove that real projective spaces  $\mathbb{R}P(n)$  are orientable for odd  $n$  and non-orientable for even  $n$ .
- 39.** Prove that complex projective spaces  $\mathbb{C}P(n)$  are orientable.
- 40.** Prove that there exists a non-vanishing smooth vector field on any odd-dimensional sphere  $\mathbb{S}^{2n-1}$ .
- 41.** Prove that the group  $SL(2, \mathbb{R})$  of  $2 \times 2$  matrices with determinant one is homotopy equivalent to the circle.