MATH 527: GEOMETRY/TOPOLOGY I

FALL 2006

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FINAL EXAM; Tuesday December 19, 2006

In order to receive perfect score you need to solve two problems from each section.

SECTION 1

1.1 Consider a figure in the plane in the shape of a thick letter T. Identify pieces of the boundary by horizontal and vertical translations.

Prove that the resulting space is a surface and identify it with one of the surfaces from the standard list.

1.2 Prove that the complement to the union of any two disjoint simple closed curves (i.e. homeomorphic images of the circle) on the torus is not connected.

1.3 Prove that any covering of the Klein bottle is homeomorphic to one of the following: plane, open cylinder, open Möbius strip, torus or Klein bottle.

Give examples of non-trivial coverings for each case.

1.4 Prove that the fundamental group of a Lie group is abelian.

SECTION 2

2.1 Prove that for any connected differentiable manifold M the group of C^∞ diffeomorphisms acts transitively, i.e. for any $x, y \in M$ there exists a C^∞ diffeomorphism $f : M \to M$ such that $f(x) = y$.

2.2 Prove that the group $SL(3, \mathbb{R})$ of 3×3 matrices with determinant one is not simply connected.

2.3 Construct on the projective plane a Morse function (i.e a smooth function all of whose critical points are not degenerate) with three critical points.

2.4 Assume the following statement: complement to the image of a compact surface under an embedding into $\mathbb{R}^3$ consists of two connected components.

Prove that there is no embedding of Klein bottle to $\mathbb{R}^3$.