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**Katok, Anatole (1-PAS)**

**Fifty years of entropy in dynamics: 1958–2007.**

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This is an attractive overview of the role played by “entropy” in the study of dynamical systems since its discovery by Kolmogorov at the end of the 1950s. Several features make it a particularly interesting read. First, the author has been an active participant in the field, often very close to the most exciting and influential developments. Second, the article does not attempt to provide a (potentially indigestible) complete history shaped by time, but instead an overview exposing the mathematical story, taking full advantage of later insights and developments. Third, it is written with candour (for example, one of the many interesting footnotes records the extent to which the disjointness paper of Furstenberg in 1967 was underestimated), with a vision of great extent in time and breadth in topic, with much detailed knowledge (for example, the ergodic-theory aspect of Wiener’s monograph of 1958 was new to me), and with great authority (both mathematical and personal: the author interacted directly with many of the key contributors both in the “Moscow school” and later in the United States). The breadth of the pervasive influence of entropy is reflected in the main sections: Early history of entropy; Isomorphism problem; Topological entropy, variational principle, symbolic dynamics; Entropy and nonuniform hyperbolicity; Entropy as a source of rigidity; Rigidity of positive entropy measures for higher-rank abelian actions.

Reviewed by *Thomas Ward*

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