# ERGODIC THEORY FROM HUMSAN TO VIENNA (1965–1997)

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*This text represents a slightly edited version of a set of notes for the talk at the opening day of the rigidity conference at the Erwin Schroedinger Institute in Vienna in February 1997 organized by Gregory Margulis, Klaus Schmidt and myself. Naturally, it reflects my point-of-view at the time which may not coincide with the present one.*

I will mostly concentrate on the earlier part of the period. I will not make an attempt to assess the mathematical developments during the last decade or so and restrict myself for that period to comments only on "organizational" developments.

## 1. Ergodic theory at 1965 (from the Moscow vantage point)

Basic entropy theory has taken shape (Kolmogorov, Sinai, Rokhlin, Parry, Adler) The isomorphism problem for Bernoulli shifts is very much on the agenda. The top achievement in that direction was Sinai weak isomorphism theorem. Mechalkin example.

Basic results of ergodic theory of hyperbolic dynamical systems (mostly Anosov and Sinai): theory of stable and unstable manifolds, absolute continuity, "Hopf argument" for ergodicity, K-property. Development of topological aspects of hyperbolic dynamics by smale and his associates. Kushnirenko inequality. Introduction of topological entropy by Adler, Konheim and McAndrew.

Theory of Gaussian dynamical systems; Gaussian systems as a source of examples (Kolmogorov, Girsanov, Vershik) mostly unsuccessful attmpts to use the Gaussian paradigm as a basis for a general theory (Sinai; my own students' work)

New trends: Revival of interest to orbit equivalence and relations to operator algebras (Kirillov, going back to von Neumann but apparently not aware of the more recent work of H.Dye). Rediscovery of combinatorial constructions (Oseledec; Katok,Stepin) study of simple extension of the rotation and interval exchange transformations which later grew into the periodic approximation method). Stepin's counterexample to the "group property" for the asymptotically cyclic group. Role of F.A.Berezin. Chacon example. Its influence of later work of Ornstein.

See the note "The 1965 Humsan school in ergodic theory" for an account of this pivotal event.

## 2. Some of the immediate consequences of Humsan

a.*Humsan proceedings.* Composition of the proceedings (Uspehi, 1967, N5). The definitive account of basic entropy theory by Rokhlin. Still one of the standard sources of the subject.

An attempt of synthesis of ergodic theory and hyperbolic dynamics (Anosov and Sinai with a considerable participation of Plykin). This is the only joint paper by the two great mathematicians. Their approaches were never completely reconciled. Anosov always looked at the subject as development of the classical theory of differential equations enriched by ideas and insights from topology, whereas for Sinai the sources of inspiration and insight were in the theory of probabilty and later more and more in mathematical physics.

An account of the new theory of periodic approxinations by Katok and Stepin (see below).This was the first major item in the "combinatorial" approach to egrodic theory.

The paper by A. Kouchnirenko on "slow" entropy, the first succesful attempt to distinguish zero entropy transformations by the (subexponential) growth type invariants.

Kirillov's account of his program of sythesis of egrodic theory, operator algebras and group representations. While seemingly without any striking direct concecuences it is related with two major later developments: (i) Zimmer's realization of the "Mackey program" and "the thoery of orbit equivalence (Krieger, Connes, Vershik)

A short note by Uzvinsky (supplement to Rokhlin's article) on entropy of group automorphisms.

A short note by Margulis (supplement to Anosov-Sinai) on fundamental group of three–dimensional manifolds carrying an Anosov flow.

b.*From combinatorial constructions to the systematic theory of periodic approximations.* Tell the story of the "triple" paper in the Humsan proceedings which became "Approximations in ergodic theory" Collapse of the Gaussian paradigm; Counterexamples maximal spectral type does not dominate its convolutions (Stepin); simple singular spectrum for the smooth flows on the torus (Katok). Jacobs popularizes the work in the West.

c.*Entropy of smooth dynamical systems* (the future "Pesin formula") Kouchnirenko's estimate from 1965 was the starting point of development. Multiplicative ergodic theorem did not quite exist yet (Oseledets proved it in 1966 or 67 but was conjectured together with the entropy formula by Sinai. Bernstein's attempt at Humsan (tell the

story). Proof of one–sided inequality by Margulis about 1968. Tell the later story here: Pesin, Mane, (Millionchshikov); Ruelle, Ledrappier.

d.*Isomorphism of Bernoulii shifts*: Bernstein and Kazhdan find new cases of isomorphism after Mechalkin and Blum-Hansen. This was perceived as the central problem in "pure" ergodic theory. Tell the story of Zaidman's attempt around 1966.

## 3. Shift in leadership (1969-1975)

Russian school(mostly Moscow and also Leningrad) hold undisputed leadership in the development of ergodic theory between the discovery of entropy by Kolmogorov in 1957 and the proof of isomorphism of Bernoulii shifts with equal entropy by Ornstein in 1968. The latter result came as a shock to Moscow. The "philosophy" of Ornstein's approach was not directly absorbed and the efforts shifted from the "pure" ergodic theory to applications, primarily to smooth dynamics and statistical mechanics. Lesdership in the development of pure ergodic theory passed to Ornstein, Furstenberg and their "schools" (Furstenberg's pioneering disjointness paper which at the time I (and probably everyone at Moscow) grossly underestimated, almost ignored.

In the development of hyperbolic dynamics something resembling the opposite (albeit not nearly as dramatic) shift happened. The original development (or rather synthesis) of hyperbolic dynamics in the sixties has two roots: algebraico-topological (Smale; compare with his comments) and analytic-geometric (Anosov, Sinai and to a lesser extent Arnold, building upon ideas of Kolmogorov and E.Hopf). While topological ideas of Smale and his followers were quickly absorbed and developed in Moscow (examples: "three papers..." , 1974 collection of translations in hyperbolic dynamics with extensive comments by Anosov and myself, Russia translation of the Nitecki book with additions and comments by Alexeyev and myself), Smale and most of his circle (with the sole remarkable exception of Bowen) were slow to absorb the probabilistic and analytical insights of the Moscow group. This resulted in a relative stagnation in the Smale group after the remarkable burst of activity in the late sixties (Hyperbolic sets, omega-stability, structiral stability, Franks' thesis). Again Bowen was an exception, producing remarkable work throughout the last ten years of his tragically short life but he did not come up with big original ideas. Rather he was developing with great power and elegance ideas introduced in the first place Smale, Sinai and Ruelle. This comment is not meant to diminish Bowen's contributions. The form he gave to the most of ergodic theory of hyperbolic systems instantly made his work classical. It

the last few yars of his life Bowen started exploring new subjects and made major insights which would undoubtedly developed into really great work has he lived longer.

Thus the main developments in hyperbolic dynamics during the seventieth, namely (i) the introduction of the thermodynamical formalism (Sinai and Ruelle),(ii) the development of non–uniformly hyperbolic dynamics (mostly Pesin) and (iii) the parameter exclusion method for proving existence of absolutely continuous invariant measures of the interval (Yakobson), (iv) hyperbolic sets in the problems of selectial mechanics (Alexeyev; also mention Conley) took place outside of Smale's original circle, even though Bowen made central contributions to the first of these developments.(Discuss variational principle here) Possibly also speak about the use of multiplicative egrodic theorem in the proof of super–rigidity by Margulis.

*Moscow seminars in the seventies.* The university seminar run by Sinai and Alexeyev and the Steklov (later CEMI) seminar run by Anosov and Katok. Describe the mode of operation. Mention some of the work which was developed and discussed there. Alexeyev (quasirandom systems and application to celestial mechanics), Anosov-Katok (smooth approximations) Brin-Pesin (partially hyperbolic systems), Bunimovich (hyperbolicity and ergodicity in billiards; tell the story of the "stadium"), Pesin (non–uniformly hyperbolic systems), Margulis (normal subgroups in lattices), Katok (flows on surfaces: monotone (Kakutani) equivalence),Yakobson, Kocergin, Satayev.

*Significant work in other areas (1968-78).*

Smooth approximation method (Anosov-Katok and further development by Herman and his circle)

Furstenberg's proof of Szemeridi Theorem and developments.

Herman's proof of Arnold conjecture and related developments.

## 4. Important meetings (1975-85)

Jerusalem 1975. Kakutani equivalence theory. Tell the story of parallel development. Vershik's ideas. How Ornstein's ideas were finally assimilated in Moscow.

Warsaw conference, summer 1977. The "East" meets the "West". See http://www.math.psu.edu/katok_a/pub/Warsaw-Szlenkconf.pdf

Warwick meetings (check dates) and the founding of Ergodic Theory and Dynamical Systems journal, the first specialized journal devoted to publications in the area and still the prime publication in the field.

Maryland 1979-80. Teichmuller theory, flows on surfaces and interval exchanges.

Berkeley MSRI. 1983-84 Completion of the first big work on "geometric rigidity": classification of higher–rank manifolds oof non–negative curvature by Ballmann, Brin, Burns Eberlein and Spatzier with a significant use of dynamical methods. Ledrappier-Young work on entropy, Lyapunov exponents and dimension.

## 5. Some events since 1985

a. Growth of the dynamics community in Latin America. Unprecendented size of dynamical establishment at IMPA. Big conferences 1981 (opening of the new IMPA building), 1989,1993, 1997 (projected). Rebirth of the dynamics group in Uruguay; 1995 congress; developments in Mexico; role of the Trieste center.

b.Florishing and partial decline of the Warwick center.

c.Great contributions made by young French mathematicians trained in geometry, Lie thery and mathematical physics.

d.Several MSRI programs related to dynamics and ergodic theory.

e.Great increase in the number, size and frequency of dynamics meetings. Two interesting tendencies. Big "congress"–type meetings (Rio, Porto 1992, Evanston 1991 Montevideo) and in the US regional conferences, often regular: (Midwestern, Penn State-Maryland, Southeastern)

f.Rigidity conferences. Close interaction between dynamics, geometry and Lie theory communities. Berkeley MSRI 1984, Caltech 1985, Caltech 1988, Boulder 1989, Northwestern 1990, Penn State 1991, Berkeley MSRI 1992, Luminy 1994, Penn State 1994, Warwick 1995, Oberwolfach 1996, Vienna 1997.