Reflections on the Program in Ergodic theory and Dynamical Systems at MSRI in 1983-84 and its aftermath

by A.Katok

The year-long program during the second year of MSRI (still operating in its temporary quarters on campus) was the central event of the early eighties for several rapidly growing and changing fields. Various dynamical disciplines (ergodic theory, topological dynamics, smooth dynamical systems, Hamiltonian dynamics) had been developing along mostly separate paths until the early seventies. During approximately 15 years preceeding the MSRI event the dominant feature was the development of big structural theories leading to classification or quasi-classification (a fairly comprehensive description) of substantial classes of dynamical systems, often axiomatically defined. These developments include the Ornstein isomorphism theory and "Kakutani equivalence" theory in ergodic theory, the structure theories for various classes of minimal systems and their extensions developed by Furstenberg and his school, hyperbolic sets and Anosov systems (Alexeyev, Anosov, Bowen, Ruelle, Sinai, Smale and many others), "Pesin theory" of smooth dynamical systems with non-zero Lyapunov characteristic exponents and, to a large extent, the KAM (Kolmogorov-Arnold-Moser) theory of preservation of invariant tori in Hamiltonian systems.

It is fair to say that by the time of the MSRI program most of those theories had already achieved a measure of conceptual understanding of principal structural components involved and the stage was being prepared for two new developments which became the dominant themes of dynamics in the eighties, namely extensive use of new general methods and paradigms in the study of various more or less concrete classes of dynamical systems and much more extensive interaction than before between various branches of dynamics on one hand and between dynamics and other mathematical disciplines on the other. The MSRI program in Ergodic theory and Dynamical systems played a very large role in both stimulating and accelerating those trends where they had already been present and in initiating new ones. Let me try to illustrate this general assesment by several examples.

One of the most impressive and fruitful developments involving interplay between several branches of dynamics (ergodic theory, smooth dynamical systems) and other mathematical disciplines (geometry, Lie groups) was a collaborative effort of W. Ballmann, M. Brin, K. Burns, P. Eberlein and R. Spatzier which led to essential understanding of the structure of compact and finite-volume Riemannian manifolds of non-positive curvature and strongly influenced the development of the idea of geometric rigidity whose roots csn be found in earlier works of Berger, Mostow and Margulis. Initial stages of this work which involved mostly an interplay between dynamics and differential geometry were undertaken by Ballmann, Brin, Eberlein and Spatzier during 1982-83. They provided crucial insights and groundwork for a big structure theorem. It was proved in the spring of 1984 independently and by completely different methods by Werner Ballmann in Bonn and by two MSRI postdoctoral fellows in the Ergodic theory and Dynamical systems program, Keith Burns and Ralf Spatzier. While Ballmann's approach to the problem may be viewed as a very brilliant shortcut based on some earlier work of Marcel Berger, Burns and Spatzier took a more fundamental and potentially more fruitful path by developing a generalisation of the theory of Tits buildings and integrating it with input from dynamics and differential geometry. The impact of their work which appeared in the form of two major articles in Publications IHES, the sister institution of MSRI in France, is far from having been exhausted. In particular, it played a major role in shaping up a research program in geometric rigidity which was one of the principal stimuli of setting up the MSRI program in Ergodic Theory, Lie groups and applications scheduled for 1991–92. Among specific developments based on the Burns-Spatzier theory one should mention a recent work of Thorbergsson on the classification of isoperimetric embeddings into Euclidean space.

It is interesting to point out that while both Burns and Spatzier received their Ph. D.'s from the University of Warwick in England (although I was Spatzier's co–advisor and de facto advisor) they are both currently tenured in major U.S. universities. Burns is an associate professor at Northwestern, and Spatzier is at Stony Brook and probably moving to the University of Michigan. Their collaboration was preceeded by Spatzier's review of the state of the subject in a series of lectures at MSRI in the fall of 1983 undertaken on my initiative and resulting in a very beautiful survey paper. After those lectures a number of MSRI members became interested in the interaction between dynamics and differential geometry, and Burns became directly involved in a very successful collaboration with Spatzier.

One of the highlights of dynamics in the seventies was the Pesin entropy formula which basically tells that all stochastic behavior with respect to an absolutely continuous invariant measure in a smooth dynamical system comes from the local hyperbolic behavior. It also gives a precise quantitative expression for the degree of stochasticity. After this landmark work came Ruelle's inequality showing that the Pesin expression gives an upper bound for the degree of stochasticity (entropy) with respect to an arbitrary invariant measure. A very natural and crucial question is to find out for what measures this upper bound is achieved and to interpret the defect between the sum of the positive Lyapunov exponents and the entropy which arises in general. This question was attacked by the French mathematician Francois Ledrappier shortly before he was invited as one of the senior participants of the Ergodic Theory and Dynamical Systems program at MSRI. He proved a converse to the Pesin formula in the case of non-zero Lyapunov exponents. While at MSRI Ledrappier developed a collaboration with Lai-Sung Young, then a postdoctoral fellow at MSRI. A native of Hong Kong, she received her Ph. D. from Berkeley under the supervision of the late Rufus Bowen. Currently she is tenured at the University of Arizona and is being sought by UCLA. Lai-Sung Young and Ledrappier achieved a complete success by establishing a converse to the Pesin formula in the general case, and describing the defect in terms of new dimension-like characteristics associated with invariant measures. The results of their work appeared as two major articles in the Annals of Mathematics.

Steve Hurder, a University of Illinois 1980 Ph. D. was an Area III postdoctoral fellow at MSRI during the 1983-84 academic year. His background and earlier results are primarily in foliation theory and its applications to topology. During his stay at MSRI he developed a deep interest in dynamics which later made him a major figure in the field as well as enabled him to make fundamental contributions to more analytical and geometric aspects of his original subject. In particular, he realized that some of the methods developed in smooth ergoodic theory may be very useful for studying characteristic classes of foliations. He gave a series of talks at MSRI which resulted, among other things, in a very fruitful long term collaboration with me. The first result of this collaboration was a major work on tempering procedures in ergodic theory and vanishing of secondary characterictic classes of foliations which appeared in the Annals. Simultaneously Hurder proved some conjectures by Sullivan related to Godbillon-Vey classes by similar methods. Later we produced a work on differentiability, rigidity and Godbillon-Vey classes which became one of the cornerstones of the "geometric rigidity" research program. It is interesting to point out that Hurder stayed at MSRI for the 1984-85 program in Operator Algebras and during that period and later made major contributions to the K-theory of operator algebras, cyclic cohomology, η-invariant and related topics. His exposure to ergodic-theoretic and dynamical methods during the previous year played a very important role in that work. Currently Hurder is a full professor at the University of Illinios at Chicago.

The crowning event of the 1983-84 activities at the MSRI was a duo of conferences in late May-early June of 1984 staged back to back and dedicated to somewhat related topics:

the conference on Group representations, Ergodic theory, Operator Algebras and Mathematical Physics in honor of G. W. Mackey (officially not a part of the Ergodic theory and Dynamical Systems program) and the workshop on Geometry, Lie Groups and Ergodic Theory organized by Bob Zimmer and myself. It seems to me fair to say that those events marked the beginning of the coherent development of a new synthetic field, which incorporates ideas and methods from Lie groups, symmetric spaces, theory of group representations Ergodic theory, smooth dynamical systems and differential geometry. In particular, the workshop included four problem sessions, dedicated to different aspects of the emerging subject. Two of those sessions resulted in extended problem lists: one on manifolds with non-positive curvature and related geometric and dynamical topics prepared by Burns and myself with substantial contributions from Ballmann, Brin, Eberlein and Osserman and another on rigidity of group actions and cocycles prepared by Hurder. Those lists which were published in Ergodic Theory and Dynamical Systems journal in 1985 contained not only problems but also motivations, background and in some cases even possibe strategies for solution. Those lists turned out to be very influential in focusing research efforts by a number of mathematicians who previously worked in different areas.

The 1984 conferences were followed by a sequence of meetings which concentrated on the progress in the program initiated at MSRI. Those include meetings at Caltech in Spring of 1985 and 1988, at the University of Colorado in May of 1989 and at Northwestern in March of 1990. The last three meetings were partially supported by NSF. Among their participants there is a core of people who came to MSRI at 1983-84 mostly as postdoctoral fellows or young visitors but it was substantially enriched by new, mostly even younger researchers. Let me give some examples of the fruitfulness of those meetings.

One of the problem sections at the MSRI workshop was dedicated to billiards. In particular, I gave a rather detailed presentation of the state of the problem of ergodicity for polygonal billiards. It was so lengthy that it annoyed Steve Kerckhoff who complained that other problems were not given an adequate hearing. However, it was exactly that discussion that introduced Kerckhoff to the problem and started a very fruitful collaboration between him, Howie Masur and John Smillie who were also among the participants of the MSRI program. It resulted in the complete solution of the problem for rational polygons in 1984. Masur's talk on the joint work was the highlight of the Spring 1985 Caltech workshop which was centered around geodesic flows and billiards. I am very happy to have annoyed Steve Kerckhoff in such a productive way.

A possibility that ergodicity and stochastic behavior may appear for geodesic flows on Riemannian manifolds very different from those of negative curvature was intensely discussed during the MSRI program. Those discussions stimulated Bob Osserman to come up with a simple but surprising example on the two-dimensional sphere, which was, however, only once dirrefentiable and had discontinuous curvature. That example was presented at the 1985 workshop and became a starting point for a very fruitful development which within several years led to C^∞ (V.Donnay) and real-analytic (K.Burns and M.Gerber) examples on the 2-sphere and all other compact surfaces, C^∞ examples on all 3-dimensional (myself) and many higher-dimensional (Burns and Gerber) manifolds as well as to the construction of ergodic systems generated by fairly realictic potential fields (Donnay and C.Liverani). The Colorado conference in particular played an important role in the development of that topic.

The 1988 Caltech conference was particularly important for the development of the geometric rigidity program which cristallyzed at MSRI. Masahiko Kanai and Ursula Hamenstadt presented their landmark works on the rigidity of optimal pinching and on geodesic flows with smooth horosheric foliations respectively. Those presentations as well as discusssions during the conference stimulated a number of young mathematicians including several students to get involved into the subject. Some of the fruits were apparent at the Colorado conference next year where in particular an extensive progress in the study of geodesic flows and Anosov system with smooth foliations was reported. This conference was remarkable by the number of very young invited speakers, five of them graduating Ph. D's (Feres, Grzegorgczyk, Hasselblatt, Iozzi, Lewis) and about as many within three years of their dissertations. In particular, Lewis reported the first major progress in Zimmer's program of classification of smooth actions of "large" groups on compact manifolds, the infinitesimal rigidity of the standard $SL(n, \mathbb{Z})$ on the n-torus, $n > 6$. This problem attracted a lot of attention and seriuos progress was achieved during the last year. First Hurder proved infinitesimal rigidity and then Lewis and myself established local rigidity. Thus, one of the central problems from the MSRI list on rigidity of group actions has been solved. Those results (the last not quite completed at that moment) were reported at the Northwestern conference. The highlight of that conference was a talk by Hamenstadt, who by that time reached a major progrees in solving probably the most important and definitely the most popular problem from the MSRI problem list on Manifolds with non-positive curvature, the Entropy Rigidity Conjecture. Her work includes, in particular, a complete solution in dimension three. Another interesting feature of that conference was participation of a new group of young mathematicians with backgrounds different of those of other participants. In particular, Kevin Corlette and Luis Hernandes presented a new proof of the Hamenstaedt result on the rigidity of pinching based on the theory of harmonic maps and found substantial generalizations of that result. By the time of the Northwestern conference it became clear that several aspects of the geometric rigidity program pursued by the participants were related with Gromov's program which embraces rigidity phenomena in different areas of mathematics Rigth now prospects for a really exciting systhesis look very good.

The next event on a somewhat larger scale is planned for March 1991 at Penn State. I hope it will both summarize a rather impressive progress achieved since the 1983-84 MSRI program and set a significant part of the agenda for the 1991-92 program in Lie Groups, Ergodic Theory with Applications to Geometry and Number Theory at MSRI.